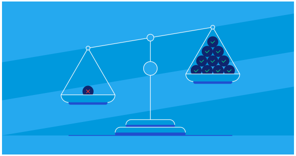
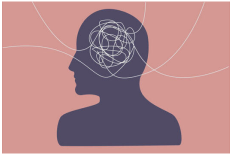
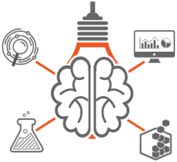
Module 5

Relations to Other Approaches for Non-Monotonic Reasoning

Relations to Formalisms for NMR

Argumentation theory has been related to a number of formalisms for NMR, among which are:

- Reasoning with (maximal) consistent subsets of premises (MCS)
- Semantics of logic programs; Answer-set programming (ASP)
- Adaptive logics
- Default logics
- Autoepistemic logics
- Input/Output logic
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- $\ldots$

Some relations to NMR have already been considered in this course:

- General patterns of NMR (e.g., the KLM postulates)
- Reasoning with MCS

Some References to Relevant Papers

Form Default Logic to ASPIC

C. Straßer and P. Pardo. *Prioritized defaults and formal argumentation.*
Proceedings of DEON'21, pp.427–446, 2021.

Form Default Logic to ABA

A. Bondarenko, P. M. Dung, R. A. Kowalski, F. Toni. *An abstract argumentation-theoretic approach to default reasoning.*
Artificial Intelligence 93, pp.63–101, 1997.

Form ASPIC to Default Logic

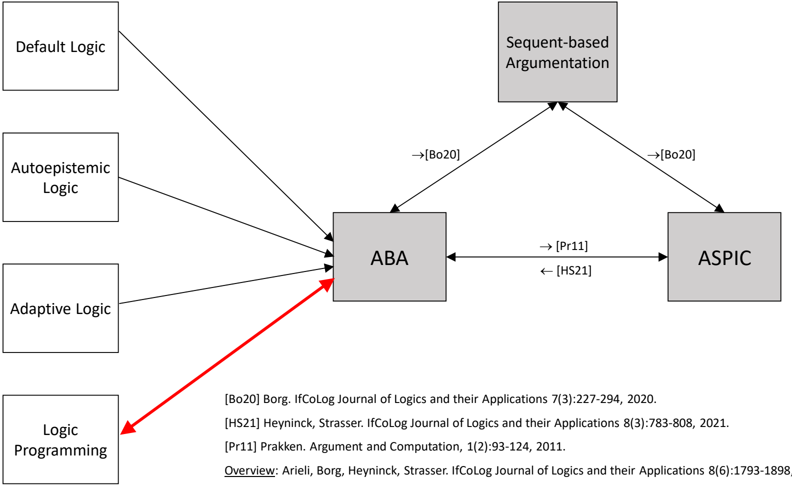
J. Heyninck and C. Straßer. *Rationality and maximal consistent sets for a fragment of ASPIC+ without undercut.*
Argument & Computation 12(1), pp.3–47, 2021.

Form Input/Output Logic to Sequent-Based Argumentation

C. Straßer, O. Arieli. *Normative reasoning by sequent-based argumentation.*
Journal of Logic and Computation 29(3), pp.387–415, 2015.

Case Study: Argumentation and Logic Programming

Recall From Module II:



[Bo20] Borg. IfCoLog Journal of Logics and their Applications 7(3):227-294, 2020.

[HS21] Heyninck, Strasser. IfCoLog Journal of Logics and their Applications 8(3):783-808, 2021.

[Pr11] Prakken. Argument and Computation, 1(2):93-124, 2011.

Overview: Arieli, Borg, Heyninck, Strasser. IfCoLog Journal of Logics and their Applications 8(6):1793-1898, 2021.

Logic Programming – A (very) Brief Overview[1]

Logic programming is a programming paradigm, based on formal logic. Major logic programming language families include Prolog, answer set programming (ASP) and Datalog.

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In all of these languages, rules are written in the form of *clauses*:

$$\phi \text{ :- } \psi_1, \ldots, \psi_n \quad (\textit{alternatively}, \phi \leftarrow \psi_1, \ldots, \psi_n)$$

and are read as logical implications: "ϕ if ψ_1 and $\ldots$ and ψ_n".

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and are read as logical implications: "ϕ if ψ_1 and $\ldots$ and ψ_n".

- ϕ is called the head of the rule, and $\psi_1, \ldots, \psi_n$ is called the body.
- Facts are rules that have no body (ϕ).

[1] Source: Wikipedia.

Types of Logic Programs

- The simplest case:
  - Positive logic programs:

    $$p \leftarrow q_1, \ldots, q_n$$

    The heads and the bodies consist only of atomic formulas.
    The rules are called definite clauses or Horn clauses.

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    The heads and the bodies consist only of atomic formulas.
    The rules are called definite clauses or Horn clauses.

- Adding capabilities of non-monotonic reasoning:

  Conditions in the body of a rule can also be negations (as failure) of atomic formulas:

  - Normal logic programs:

    $$p \leftarrow q_1, \ldots, q_n, \text{not } r_1, \ldots, \text{not } r_m$$

  - Disjunctive logic programs:

    $$p_1 \vee \ldots \vee p_k \leftarrow q_1, \ldots, q_n, \text{not } r_1, \ldots, \text{not } r_m$$

  - Extended [normal/disjunctive] logic programs:
    Literals ($l \in \{p, \neg p\}$) instead of atoms.

Negation As Failure

$p_1 \vee \ldots \vee p_k \leftarrow q_1, \ldots, q_n, \text{not } r_1, \ldots, \text{not } r_m$ may be read as follows:

"If q_i holds (for every $i = 1, \ldots n$) and r_i fails to hold (for every $i = 1, \ldots m$), then at least one of p_i's must hold (for some $1 \leq i \leq k$)."

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Example

Consider the following normal logic program:

$$\begin{array}{l} canfly(X) \leftarrow bird(X), not\ abnormal(X) \\ abnormal(X) \leftarrow wounded(X) \\ bird(John) \\ bird(Mary) \end{array}$$

We would like to infer in this case that both John and Mary can fly.

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We would like to infer in this case that both John and Mary can fly.

But if we are informed that:

$$wounded(John)$$

then now we should conclude that only Mary can fly.

Semantics of Logic Programs

$\mathcal{P}$ – a disjunctive logic program (DLP),
Atoms($\mathcal{P}$) – the set of atomic formulas that appear in $\mathcal{P}$.

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- $M \subseteq \text{Atoms}(\mathcal{P})$ *satisfies* a rule
  $p_1 \vee \ldots \vee p_k \leftarrow q_1, \ldots, q_n, \text{not } r_1, \ldots, \text{not } r_m$ in $\mathcal{P}$ iff either:
  $\exists 1 \leq i \leq n\ q_i \notin M$, or $\exists 1 \leq i \leq m\ r_i \in M$, or $\exists 1 \leq i \leq k\ p_i \in M$.
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- M is a *model* of $\mathcal{P}$ if it satisfies every rule in $\mathcal{P}$.

- The *Gelfond-Lifschitz reduct* [1] of $\mathcal{P}$ with respect to M is the disjunctive (positive) logic program $\mathcal{P}^M$, where
  $p_1 \vee \ldots \vee p_k \leftarrow q_1, \ldots, q_n \in \mathcal{P}^M$ iff there is a rule
  $p_1 \vee \ldots \vee p_k \leftarrow q_1, \ldots, q_n, \text{not } r_1, \ldots, \text{not } r_m \in \mathcal{P}$
  and $r_i \notin M$ for every $1 \leq i \leq m$.
- M is a *stable model* of $\mathcal{P}$ iff it is a $\subseteq$-minimal model of $\mathcal{P}^M$.

[1] M. Gelfond and V. Lifschitz. *The Stable Model Semantics for Logic Programming*, Proc. ICLP'88, pp.1070–1080, 1988.

Example, Continued

$$P_1 = \left\{ \begin{array}{l} \mathit{canfly}(\mathit{John}) \leftarrow \mathit{bird}(\mathit{John}), \mathit{not}\ \mathit{abnormal}(\mathit{John}) \\ \mathit{abnormal}(\mathit{John}) \leftarrow \mathit{wounded}(\mathit{John}) \\ \mathit{bird}(\mathit{John}) \end{array} \right\}$$

$M_1 = \{\mathit{bird}(\mathit{John}), \mathit{canfly}(\mathit{John})\}$ is the stable model of $\mathcal{P}_1$, since it is a $\subseteq$-minimal model of the GL-reduct

$$P_1^{M_1} = \left\{ \begin{array}{l} \mathit{canfly}(\mathit{John}) \leftarrow \mathit{bird}(\mathit{John}) \\ \mathit{abnormal}(\mathit{John}) \leftarrow \mathit{wounded}(\mathit{John}) \\ \mathit{bird}(\mathit{John}) \end{array} \right\}$$

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$$P_1^{M_1} = \left\{ \begin{array}{l} \mathit{canfly}(\mathit{John}) \leftarrow \mathit{bird}(\mathit{John}) \\ \mathit{abnormal}(\mathit{John}) \leftarrow \mathit{wounded}(\mathit{John}) \\ \mathit{bird}(\mathit{John}) \end{array} \right\}$$

$$P_2 = P_1 \cup \mathit{wounded}(\mathit{John})$$

$M_2 = \{\mathit{bird}(\mathit{John}), \mathit{wounded}(\mathit{John}), \mathit{abnormal}(\mathit{John})\}$ is the stable model of $\mathcal{P}_2$, since it is a $\subseteq$-minimal model of the GL-reduct

$$P_2^{M_2} = \left\{ \begin{array}{l} \mathit{abnormal}(\mathit{John}) \leftarrow \mathit{wounded}(\mathit{John}) \\ \mathit{bird}(\mathit{John}) \\ \mathit{wounded}(\mathit{John}) \end{array} \right\}$$

Representation of DLPs by ABFs

Goal: Given a disjunctive logic program (DLP) $\mathcal{P}$, we construct an assumption-based argumentation framework (ABF) $\mathcal{ABF}$, such that there is a one-to-one correspondence between the stable models of $\mathcal{P}$ and the stable extensions of $\mathcal{AF}$.

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Recall:

An *assumption-based framework* is a tuple $\mathcal{ABF} = \langle \mathfrak{L}, \Gamma, \Delta, \sim \rangle$, s.t.:

- $\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$ is a (propositional) logic,
- Γ is a set of $\mathcal{L}$-formulas, called the *strict rules*,
- Δ is a set of $\mathcal{L}$-formulas, called the *defeasible assumptions*,
- $\sim : \Delta \to 2^{\mathcal{L}}$ is a *contrariness operator*.

$\Theta \subseteq \Delta$ *attacks* ψ if there are $\Theta' \subseteq \Theta$ and $\phi \in {\sim}\psi$ such that $\Gamma, \Theta' \vdash \phi$.

Θ_1 attacks Θ_2 if Θ_1 attacks some $\psi \in \Theta_2$.

The Underlying Logic

All the ABA frameworks that are induced from disjunctive logic programs are based on the same core logic $\mathfrak{L}_{\mathsf{DLP}} = \langle \mathcal{L}, \vdash \rangle$, where:

- $\mathcal{L} = \{\leftarrow, \vee, \mathit{not}\}$ consists of disjunctions of atoms ($p_1 \vee \ldots \vee p_n$ for $n \geq 1$), negated atoms (*not p*), or DLP rules.
- $\mathcal{S} \vdash \psi$ iff $\psi \in \mathcal{S}$ or ψ is derived from $\mathcal{S}$ using Modus Ponens (MP), Resolution (Res) and Reasoning by Cases (RBC).

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- $\mathcal{S} \vdash \psi$ iff $\psi \in \mathcal{S}$ or ψ is derived from $\mathcal{S}$ using Modus Ponens (MP), Resolution (Res) and Reasoning by Cases (RBC).

[MP] $$\frac{p1 \vee \ldots \vee p_n \leftarrow l_1, \ldots, l_n \quad l_1 \quad l_2 \quad \cdots \quad l_n}{p1 \vee \ldots \vee p_n} \quad (\ l_i \in \{p_i, \mathit{not}\ p_i\}\ )$$

[Res] $$\frac{p'_1 \vee \ldots \vee p'_m \vee q_1 \vee \ldots \vee q_n \vee p''_1 \vee \ldots \vee p''_k \quad \mathit{not}\ q_1 \quad \cdots \quad \mathit{not}\ q_n}{p'_1 \vee \ldots \vee p'_m \vee \ldots \vee p''_1 \vee \ldots \vee p''_k}$$

[RBC] $$\frac{\begin{array}{c} q_1 \\ \vdots \\ p_1 \vee \ldots \vee p_n \end{array} \quad \begin{array}{c} q_2 \\ \vdots \\ p_1 \vee \ldots \vee p_n \end{array} \quad \cdots \quad \begin{array}{c} q_m \\ \vdots \\ p_1 \vee \ldots \vee p_n \end{array} \quad q_1 \vee \ldots \vee q_m}{p_1 \vee \ldots \vee p_n}$$

Representation of DLPs by ABFs

Definition

The assumption-based argumentation framework that is *induced* by a disjunctive logic program $\mathcal{P}$ is $\mathcal{ABF}(\mathcal{P}) = \langle \mathfrak{L}_{\mathsf{DLP}}, \Gamma, \Delta, \sim \rangle$, where: $\Gamma = \mathcal{P}$, $\Delta = \{not\ p \mid p \in \mathsf{Atom}(\mathcal{P})\}$, and $\forall p \in \mathsf{Atom}(\mathcal{P})$ $\sim(not\ p) = \{p\}$.

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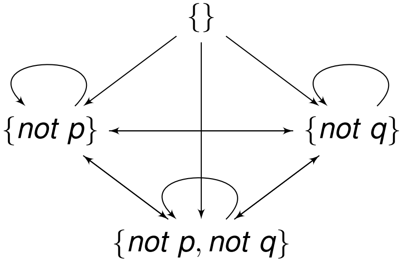
Example: $\mathcal{P} = \{p \vee q \leftarrow,\ p \leftarrow q,\ q \leftarrow p\}$

Attack diagram for the induced ABF:



For instance, *not q* attacks itself since $\mathcal{P}$, *not q* $\vdash \sim(not\ q)$, i.e., $\mathcal{P}$, *not q* $\vdash q$.
Indeed, $p \vee q \leftarrow\ \in \mathcal{P}$. By [MP], $p \vee q$. [Res] with *not q* gives p, and [*MP*] with $q \leftarrow p$ gives q.

Representation of DLPs by ABFs

Let $\mathcal{P}$ be a disjunctive logic program and $\Theta \subseteq \text{Atoms}(\mathcal{P})$.

We denote:

- $\textit{not}\ \Theta = \{\textit{not}\ \theta \mid \theta \in \Theta\}$.
- $\lfloor \textit{not}\ \Theta \rfloor = \Theta$.
- If $\Delta \subseteq \textit{not}\ \text{Atoms}(\mathcal{P})$ then $\underline{\Delta} = \text{Atoms}(\mathcal{P}) \setminus \lfloor \Delta \rfloor$.
- If $\Delta \subseteq \text{Atoms}(\mathcal{P})$ then $\overline{\Delta} = \textit{not}(\text{Atoms}(\mathcal{P}) \setminus \Delta)$.

Thus, $\lfloor \Theta \rfloor$ eliminates the leading *not* from formulas in Θ.
$\underline{\Delta}$ (respectively, $\overline{\Delta}$) takes the complementary set of Δ and removes (respectively, adds) the negation-as-failure operator from (respectively, to) the prefix of its formulas.

Theorem

- *If Δ is a stable extension of $\mathcal{ABF}(\mathcal{P})$, $\underline{\Delta}$ is a stable model of $\mathcal{P}$,*
- *If Δ is a stable model of $\mathcal{P}$, $\overline{\Delta}$ is a stable extension of $\mathcal{ABF}(\mathcal{P})$.*

Example, Revisited

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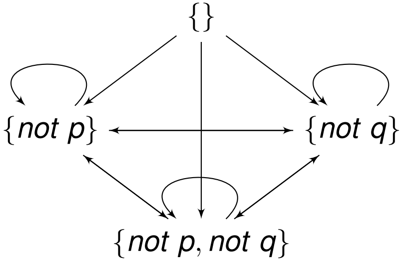

Atoms($\mathcal{P}$) $= \{p, q\}$. The stable model of $\mathcal{P}$ is {p,q}.
The stable extension of $\mathcal{ABF}(\mathcal{P})$ is $\overline{\{p, q\}} = \{\}$.

Further Notes

Theorem

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- *If Δ is a stable model of $\mathcal{P}$, $\overline{\Delta}$ is a stable extension of $\mathcal{ABF}(\mathcal{P})$.*

- The theorem was shown for disjunctive normal programs in [1].
- Caminada and Schulz [2] have shown that for normal logic programs the base logic of the ABF may contain only [MP].
- Wakaki [3] has proven similar results for extended normal logic programs.

[1] J. Heyninck , O. Arieli. *An argumentative characterization of disjunctive logic programming*. Proceedings of EPIA'21, LNCS 11805, pp.52–538, Springer, 2019.

[2] M. Caminada, C. Schulz. *On the equivalence between assumption-based argumentation and logic programming*. Artif. Intell. Research 60:779–825, 2017.

[3] T. Wakaki. *Consistency in assumption-based argumentation*. Proceedings of COMMA'20, pp.371–382, IOS Press, 2020.